1) [for all] y Y, [there exists] x X , such that f(x) = y

a) f: Z -> Z, x -> x-1

suppose y is an integer. F(x) =1

x – 1 = y (add one to each side)

x = y+ 1

thus: f(x) = f(y + 1)

= (y+1) – 1 = y

therefore f is *onto* because y is an integer.

b) f: Z-> Z, x -> 4x – 5

f(x) = y. The co-domain of y is Z(an integer). 0 Z but f(x) != 0 for an integer x. for if f(x) = 0 then:

4x – 5 = 0

4x = 5

x = 5/4

But 5/4 is not an integer so F is not *onto*.

c) f: Z-> Z, x -> x2 + 1

f(x) = y. The co-domain of y is Z(an integer). -3 Z but f(x) != -3 for an integer x. for if f(x) = -3 then:

x2 + 1 = -3

sqrt(x2) = sqrt(-4)

x = 2\*sqrt(-1) Which is not an integer therefore it is not *onto*.

2) [for all] x1, x2 X, if f(x1) = f(x2) then x1 = x2

a)f: Z->Z, x -> x-1

suppose x1 and x2 are integers such that f(x1) = f(x2) [must show that x1=x2] by definition of f…

x1 – 1 = x2 – 1, adding 1 to both sides gives…

x1 = x2, which is what needs to be shown for it to be a true one-to-one function.

b) f: Z -> Z, x -> 4x – 5

suppose x1 and x2 are integers such that f(x1) = f(x2) [must show that x1=x2] by definition of f…

4x1 – 5 = 4x2 – 5, add 5…

4x1 = 4x2, divide by 4…

x1 = x2, which is what needs to be shown for it to be a true one-to-one function.

c) f: Z -> Z, x -> x2 + 1

f is not one-to-one. (ex: x1 = 2 and x2 = -2;

f(x1) = 22 + 1 = 5 and

f(x2) = -22 + 1 = 5 but

2 != -2.)

d) f: R-> R, x -> (x + 1) / (x – 1)

If x =1 then the function cannot output a result and thus is not one-to-one. But if we suppose that x != 1 for all real numbers x then the following would work as the proof:

suppose x1 and x2 are integers such that f(x1) = f(x2) [must show that x1=x2] by definition of f…

(x1 + 1) / (x1 – 1) = (x2 + 1) / (x2 – 1) multiply each side by its denominator…

(x1 + 1)(x2 – 1) = (x2 + 1)(x1 – 1) foil it out…

x1x2 + x2 – x1 – 1 = x1x2 + x1 – x2 -1 add/subtract…

x2 + x2 = x1 + x1

2x2 = 2x1 Divide both sides by 2…

x2 = x1, which is what needs to be shown for it to be a true one-to-one function.

3)

a) Drawing for graph:

b) walks – 1) e, ed, d.

2)e, eb, b, bd, d

c) Paths – 1) a, ae, e

2) a, ab, b, be, e

3) a, ab, b, bd, d, de, e

d) trails - 1) b, ab, a

2) b, be, e, ea, a

3) b, bd, d, de, e, ea, a

e) cycles – 1) a, ab, b, be, e, ea, a

2) a, ab, b, bd, d, de, e, ea, a

3) b, be, e, ed, d, db, b

f) No there is no cycle that includes every vertex because there is no edge connecting vertex c to the rest of the graph.

4) I found the glossing over the function concepts and spending most of the time on the graphs the most frustrating when the majority of the homework was spent on the functions. The thing I found most enjoyable was the overview of the midterm and the excellent job I did on it.

5) The reason the magician can tell you your number is because when you tell him which cards your number are on you are telling him your number. This is possible because with the 4 cards he sets it up so each has a true or false associated with it and when you tell him which cards have your number you tell him the true. There can only be 15 true false combinations. (The all false is omitted because it looks less like magic if you tell him your number isn’t present and he knows it automatically.) The cards are set up as follows(he just memorizes this table and poof! Magic!):

# 1 2 3 4

1 T F F F

2 F T F F

3 T T F F

4 F F T F

5 T F T F

6 F T T F

7 T T T F

8 F F F T

9 T F F T

10 F T F T

11 T T F T

12 F F T T

13 T F T T

14 F T T T

15 T T T T